Worst-case uncertainty construction for V&V

Bálint Patartics[†], Peter Seiler[⋆], Bálint Vanek[†]

[†]Institute of Computer Science and Control (Hungary)
[⋆]University of Michigan (USA)

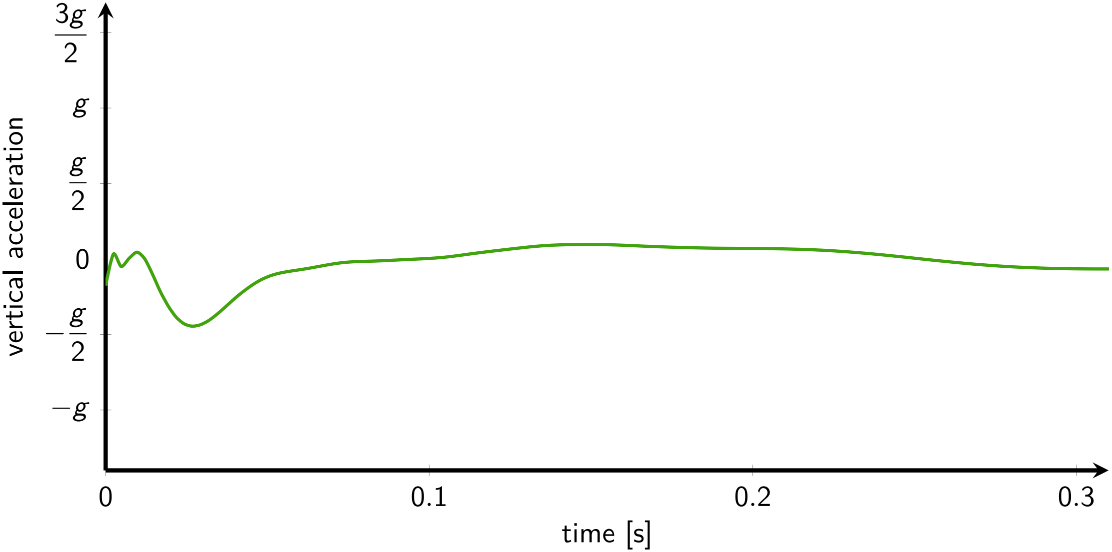
April 15, 2021

The research leading to these results is part of the EU project FLiPASED (grant agreement No. 815058).

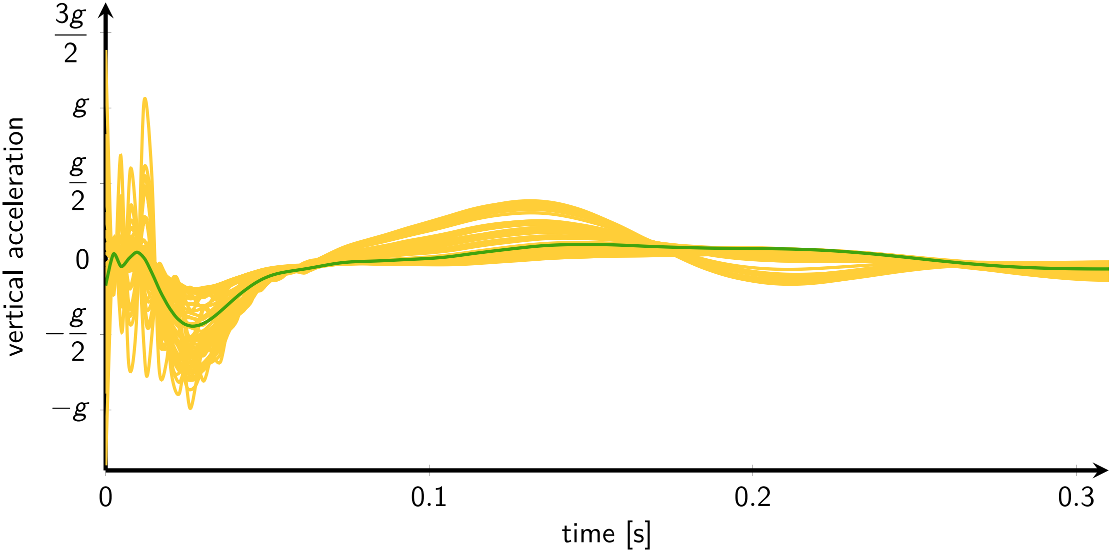
Worst-case uncertainty

Worst-case uncertainty

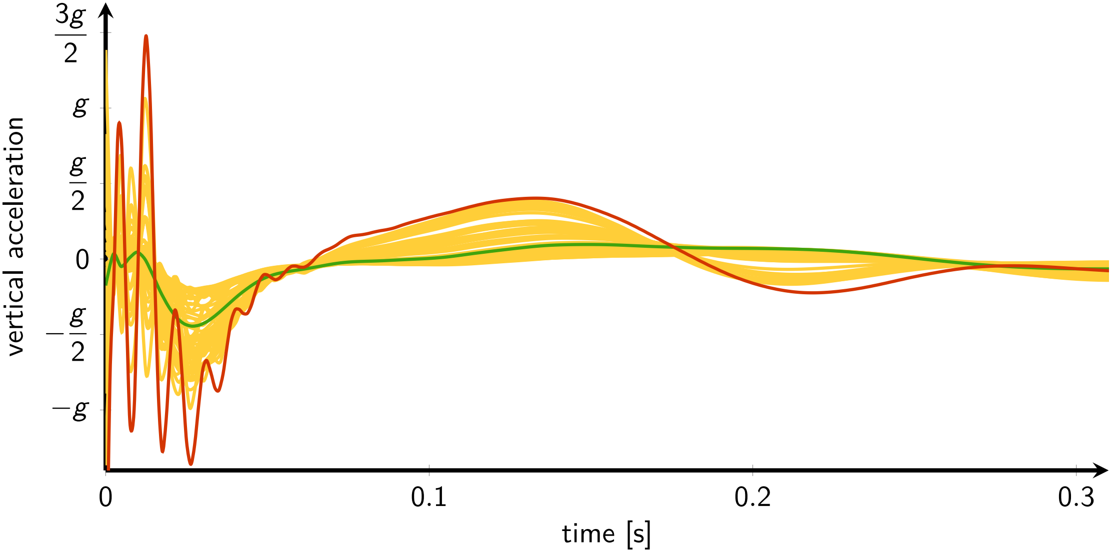
Worst-case uncertainty



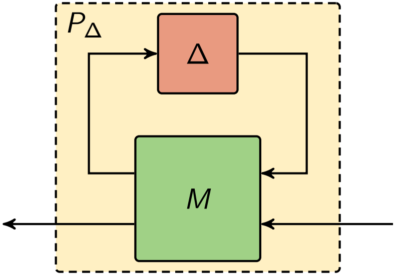
Worst-case uncertainty



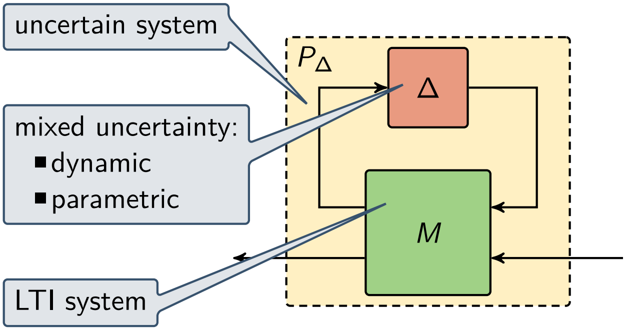
Worst-case uncertainty



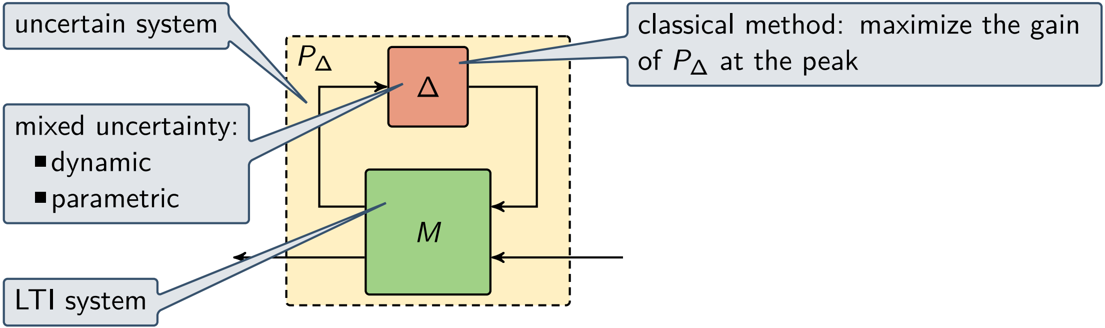
Worst-case uncertainty construction



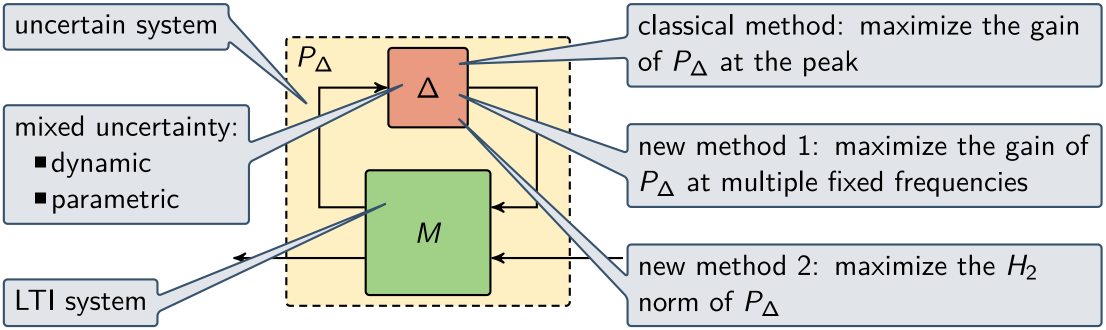
Worst-case uncertainty construction



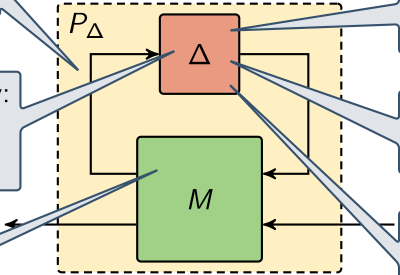
Worst-case uncertainty construction



Worst-case uncertainty construction



Worst-case uncertainty construction



uncertain system

mixed uncertainty:
- dynamic
- parametric

LTI system

classical method: maximize the gain of P_Δ at the peak

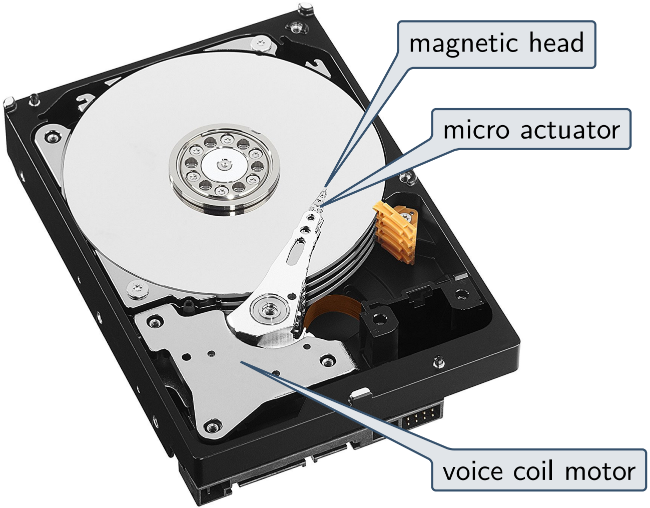
new method 1: maximize the gain of P_Δ at multiple fixed frequencies

new method 2: maximize the H_2 norm of P_Δ

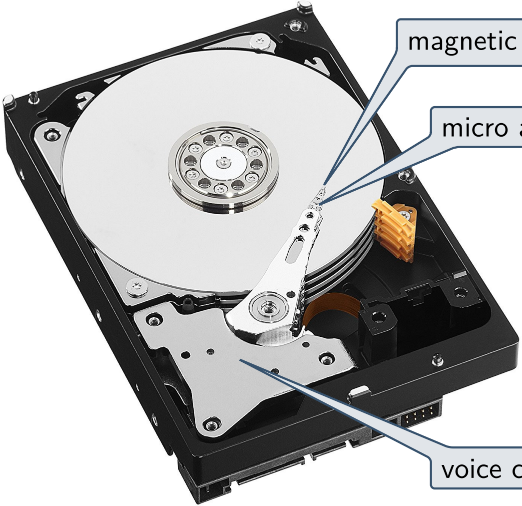
- Patartics, Seiler, Vanek. *Construction of an uncertainty to maximize the gain at multiple frequencies.* ACC 2020.
- Patartics, Seiler, Takarics, Vanek. *Worst-case uncertainty construction via multifrequency gain maximization.* Submitted to the TCST, 2020

Hard disk drive

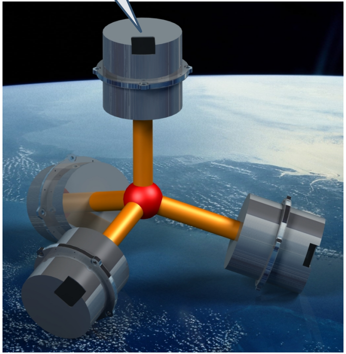
Hard disk drive



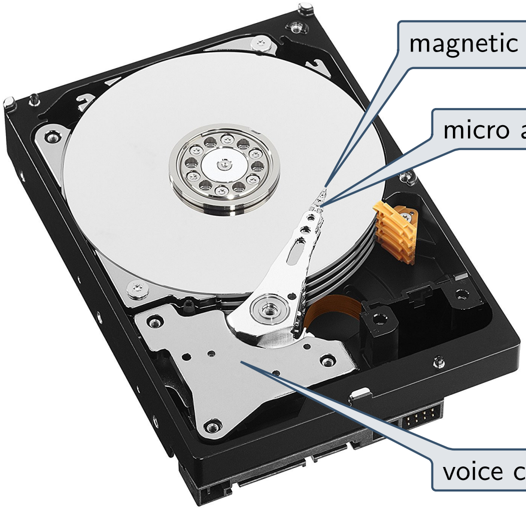
Hard disk drive



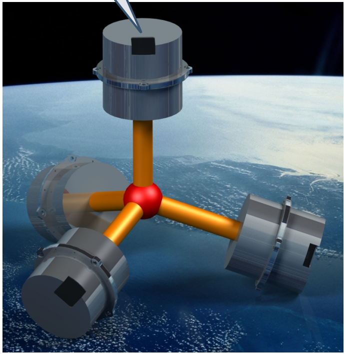
analogy



Hard disk drive

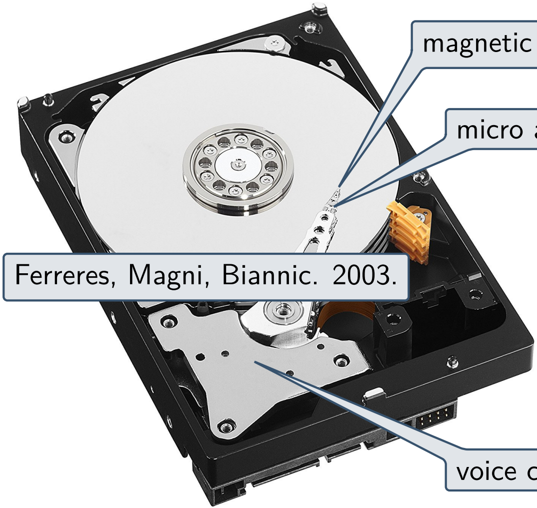


analogy



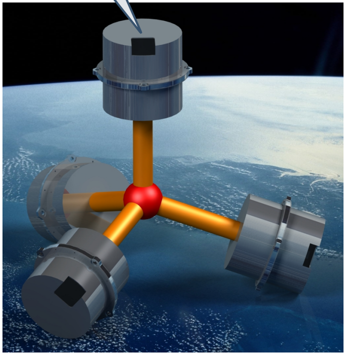
Sahar, Edlerman, Agalarian, Balabanov, Gurfil. 2017.

Hard disk drive



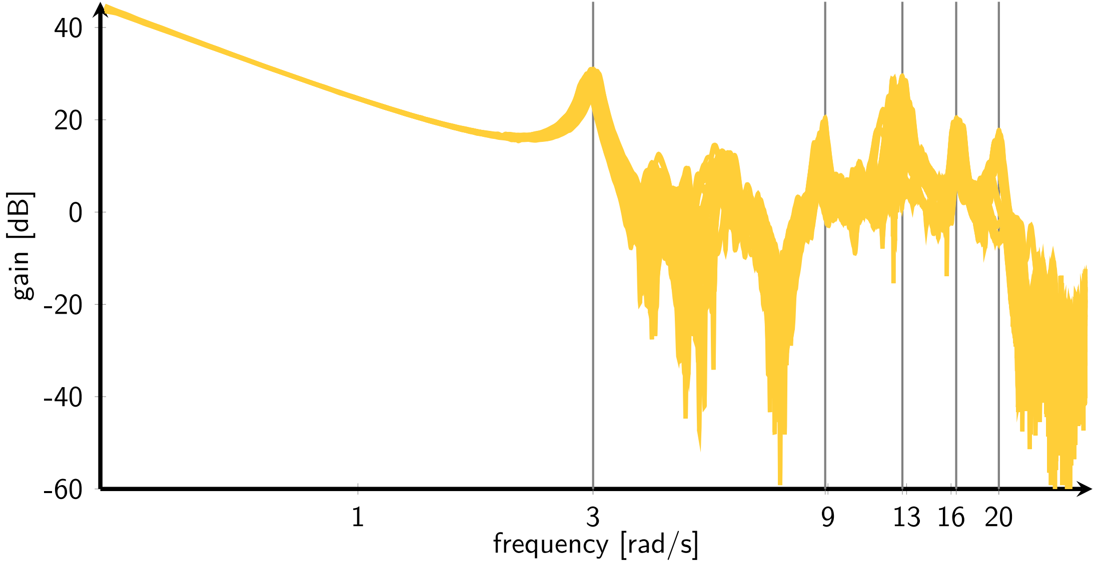
Ferreres, Magni, Biannic. 2003.

analogy

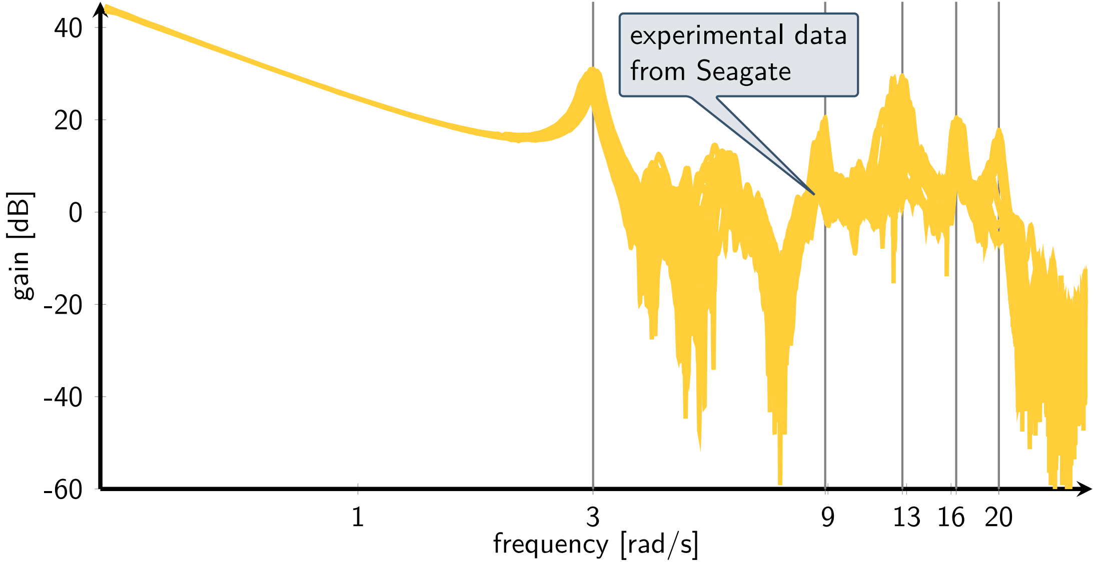


Sahar, Edlerman, Agalarian, Balabanov, Gurfil. 2017.

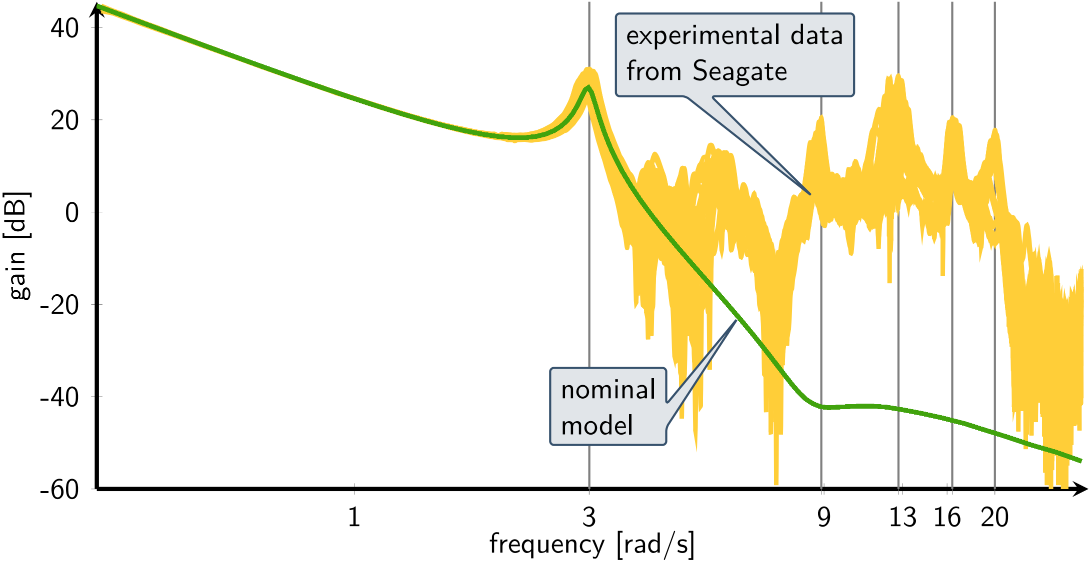
Hard disk drive



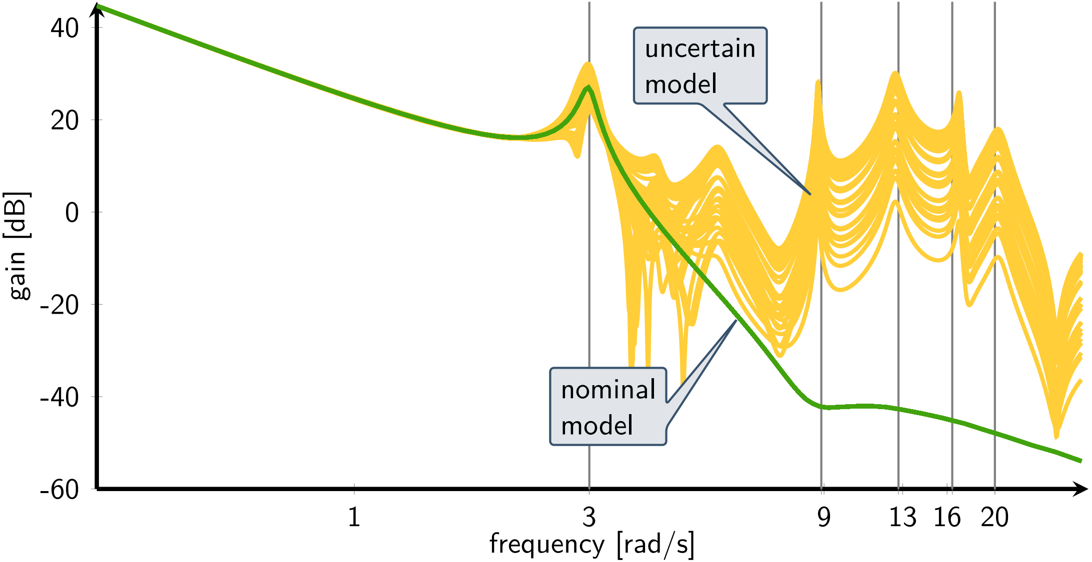
Hard disk drive



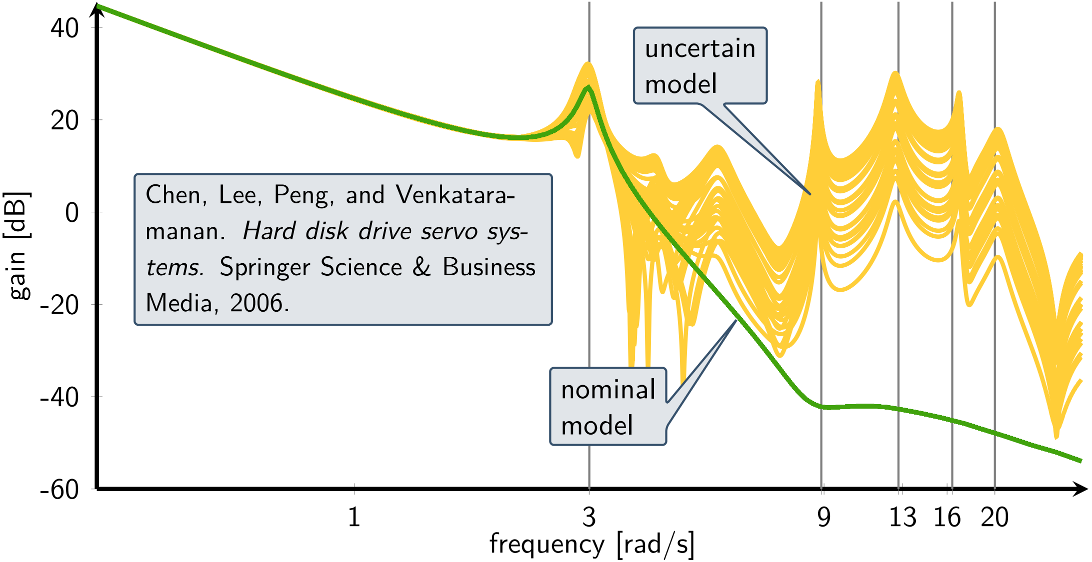
Hard disk drive



Hard disk drive

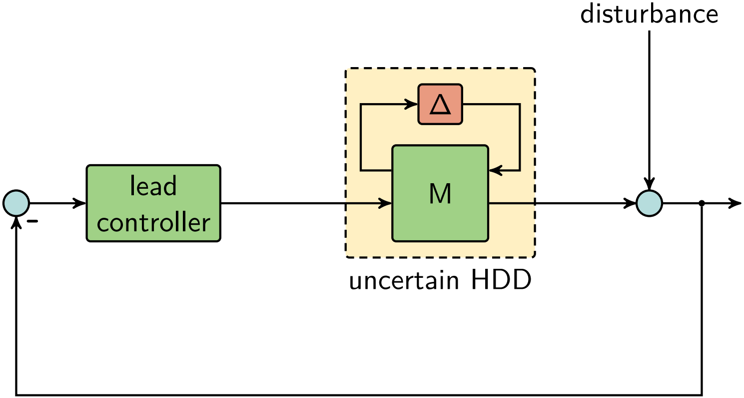


Hard disk drive

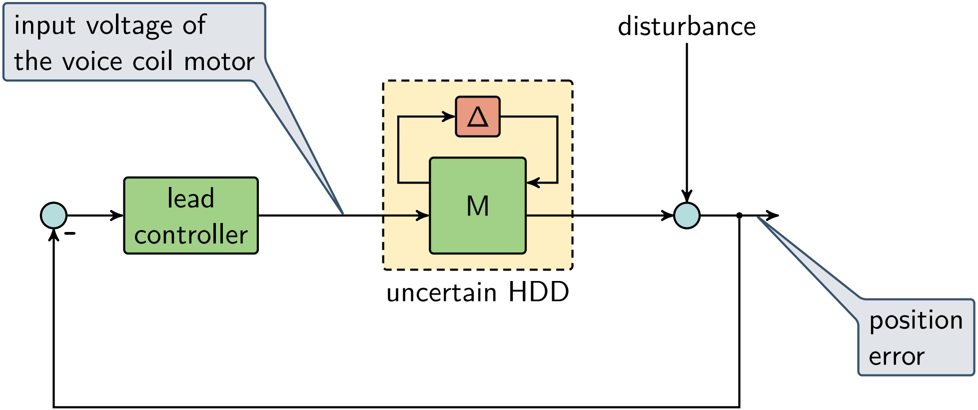


Chen, Lee, Peng, and Venkataramanan. *Hard disk drive servo systems.* Springer Science & Business Media, 2006.

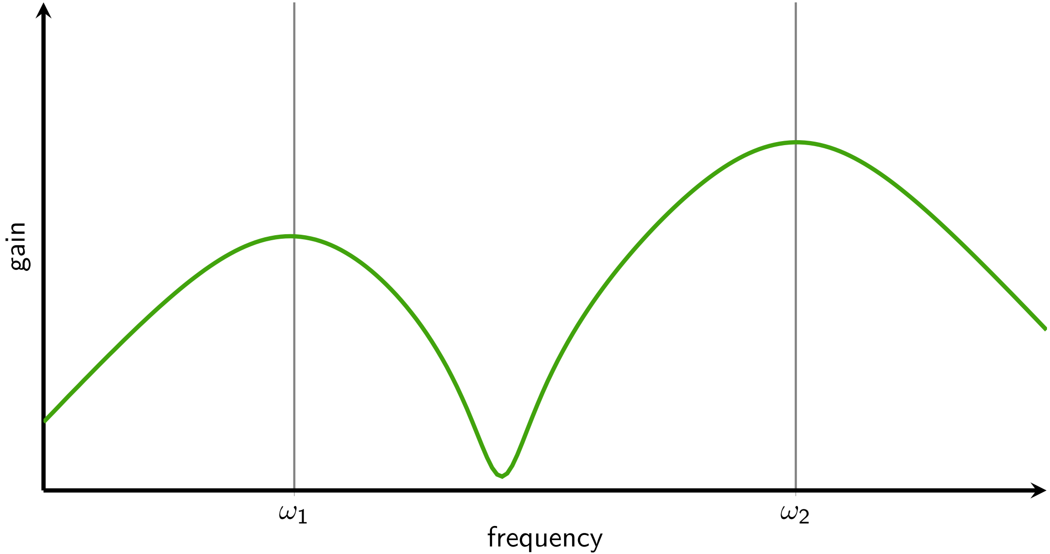
Hard disk drive



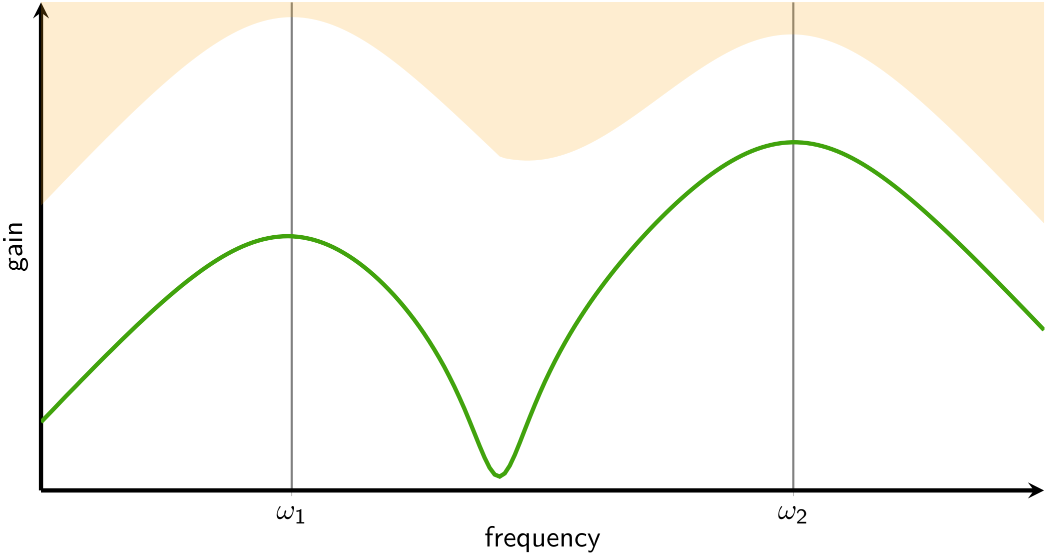
Hard disk drive



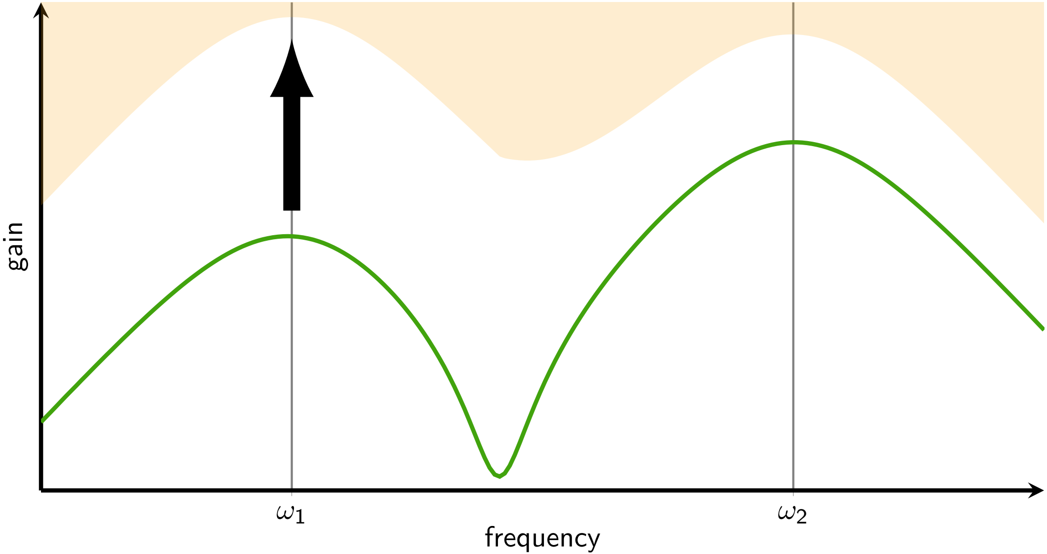
Multi-frequency worst-case



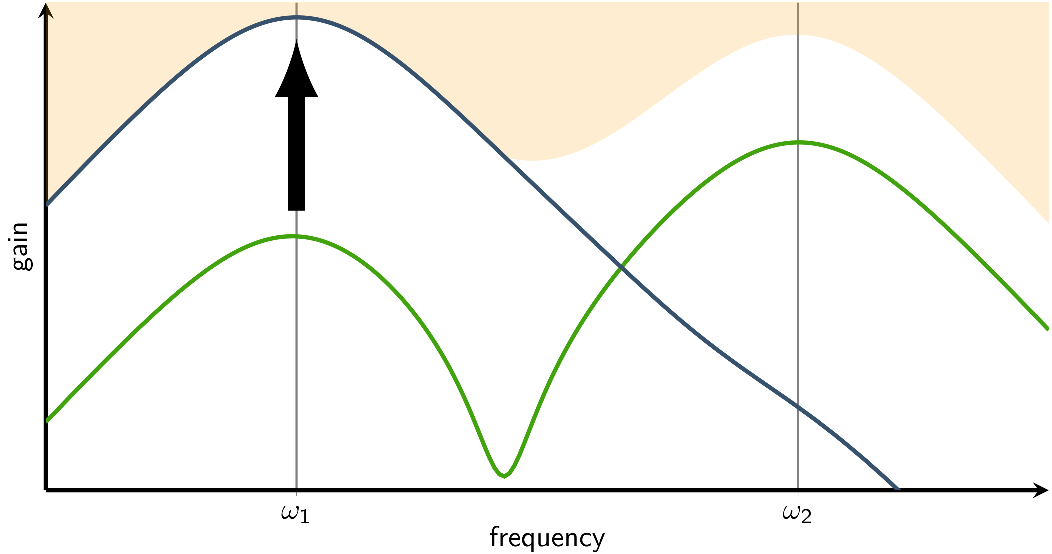
Multi-frequency worst-case



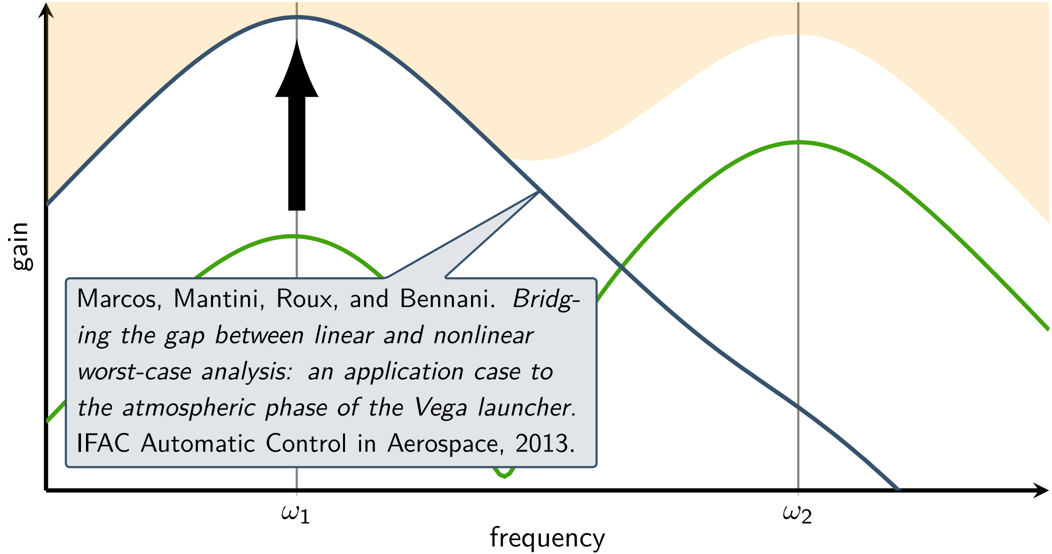
Multi-frequency worst-case



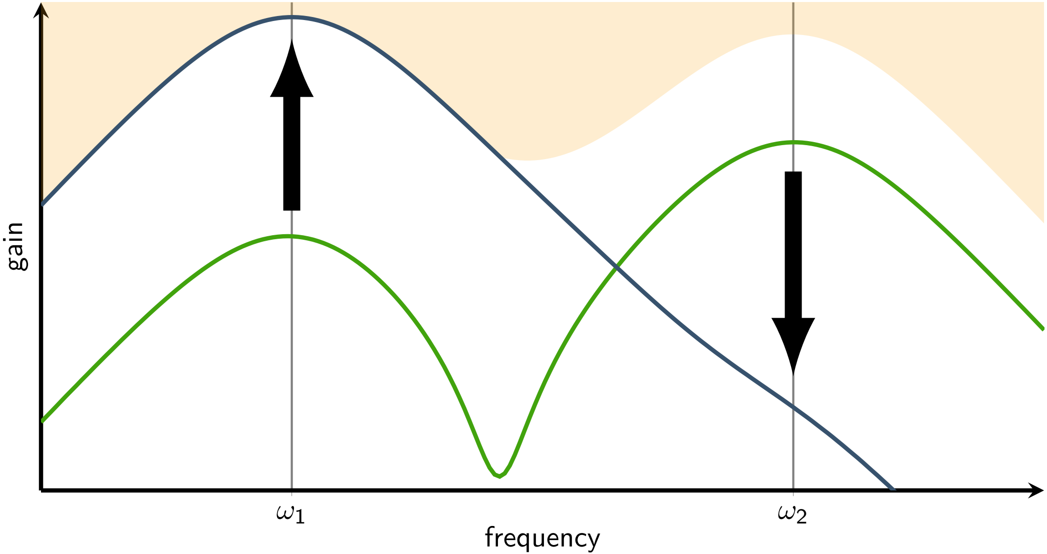
Multi-frequency worst-case



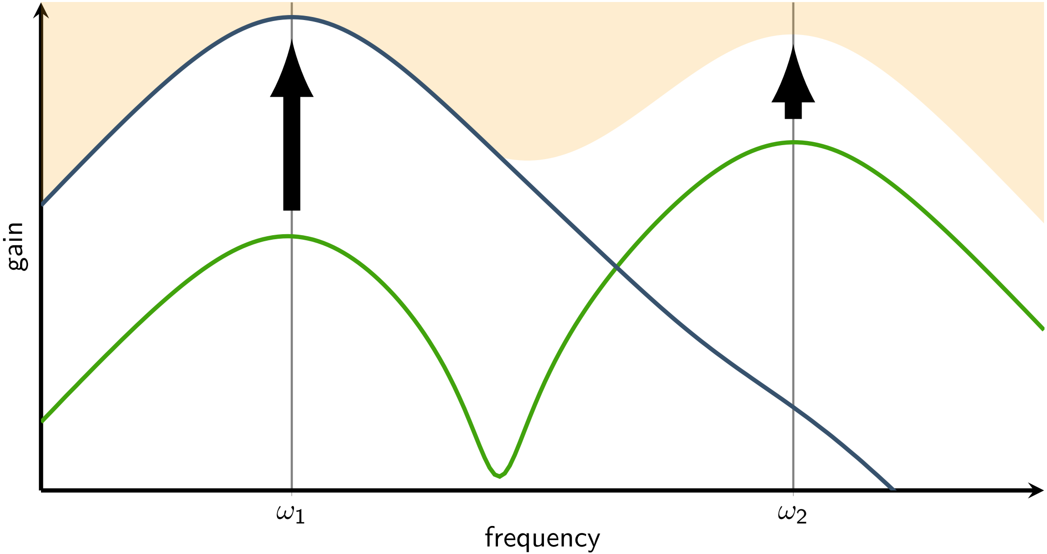
Multi-frequency worst-case



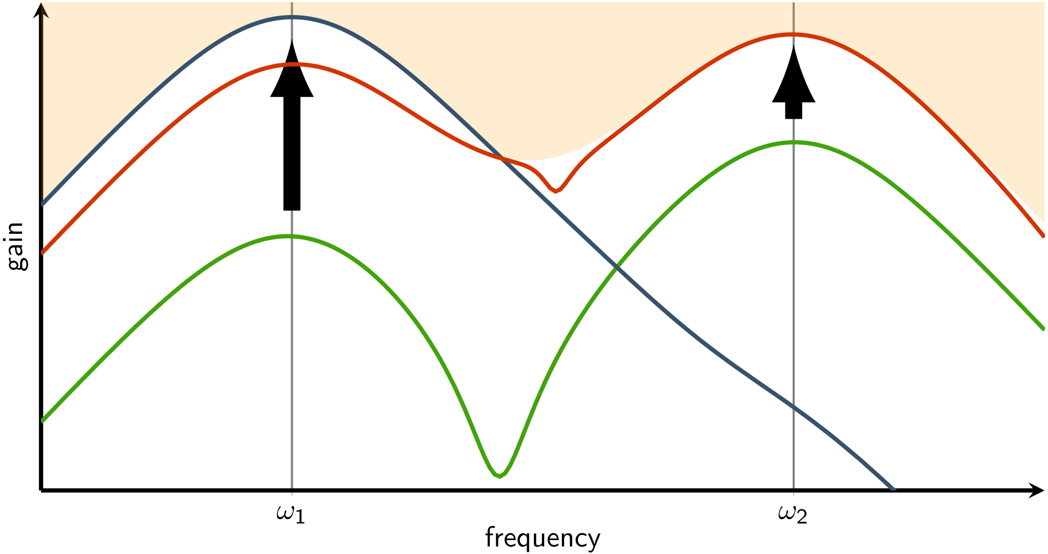
Multi-frequency worst-case



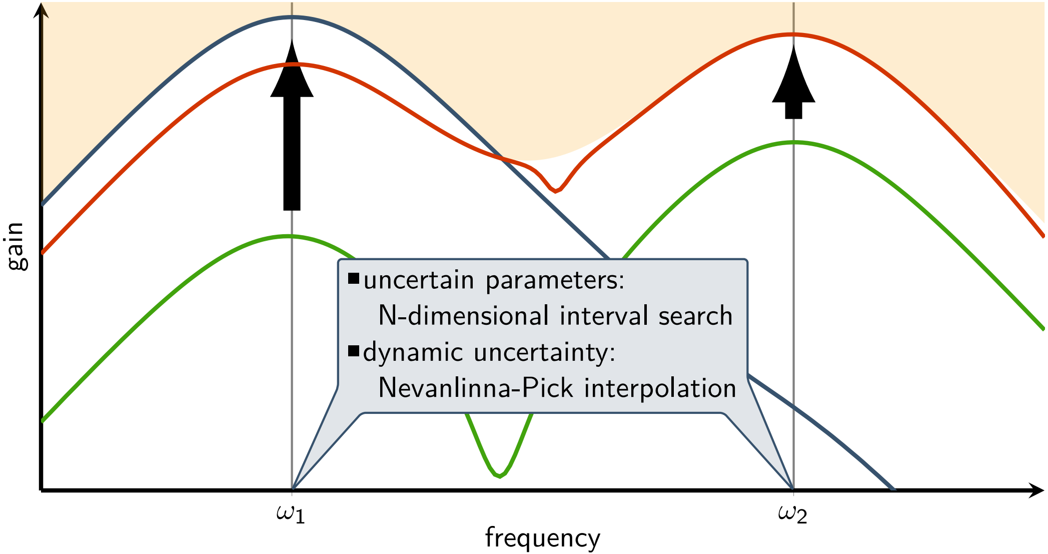
Multi-frequency worst-case



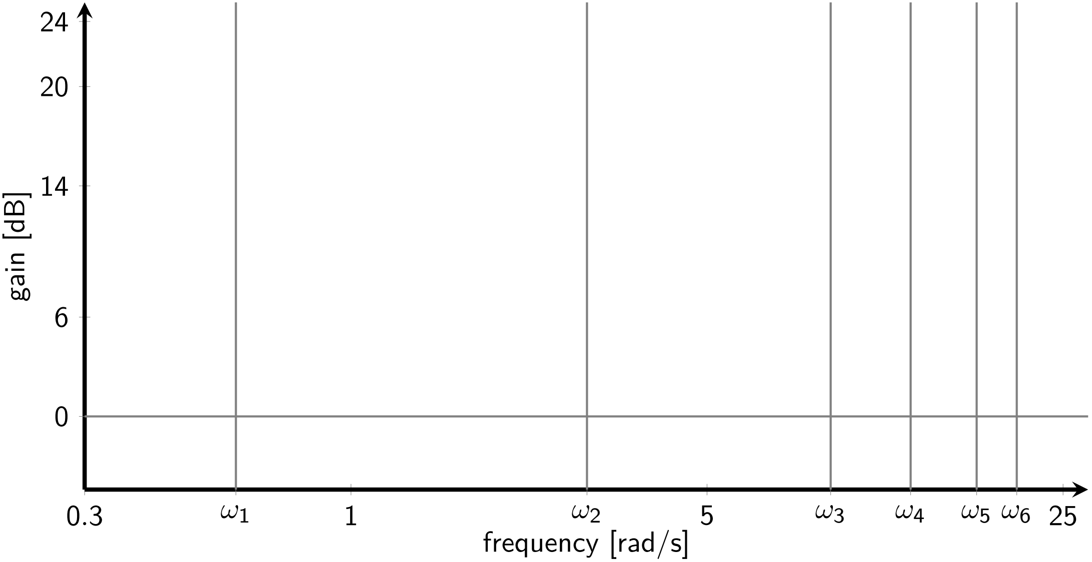
Multi-frequency worst-case



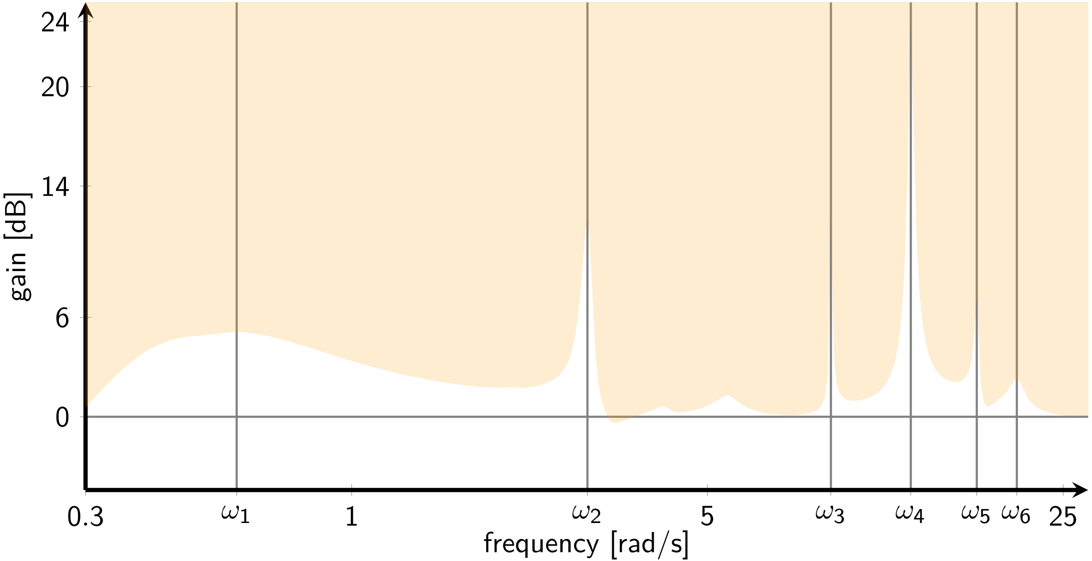
Multi-frequency worst-case



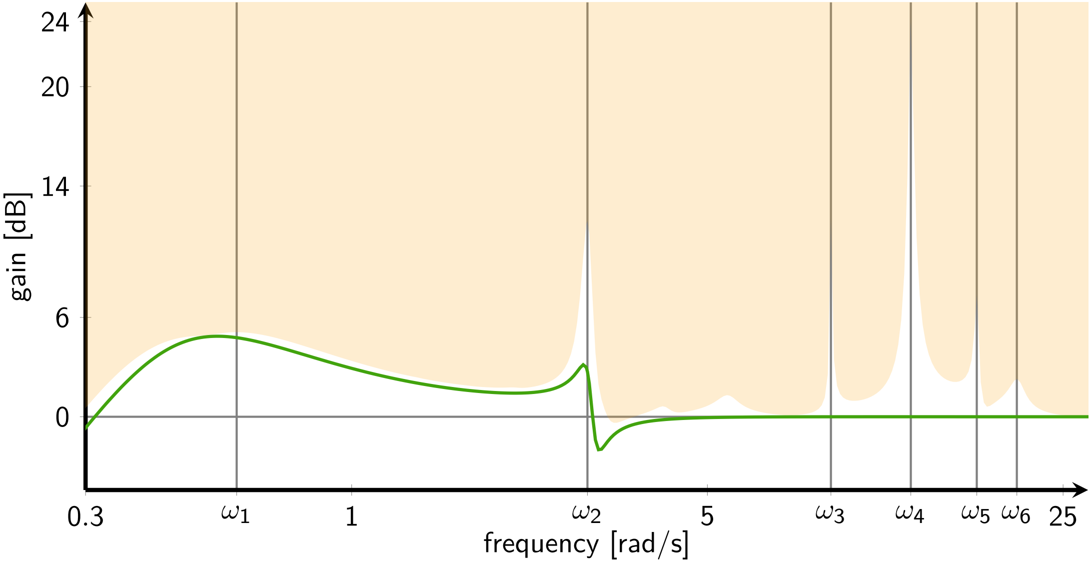
Multi-frequency worst-case



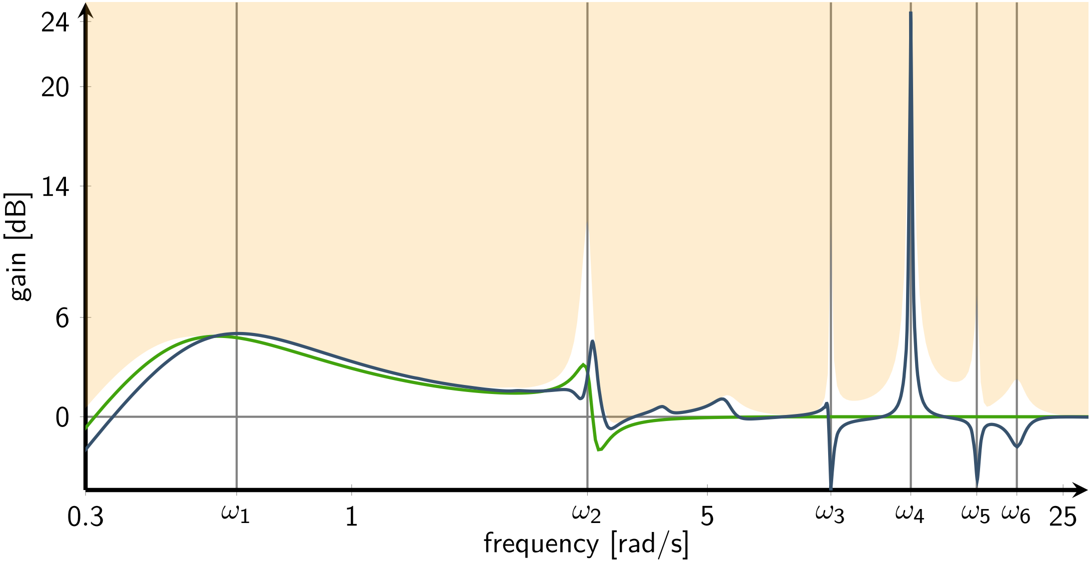
Multi-frequency worst-case



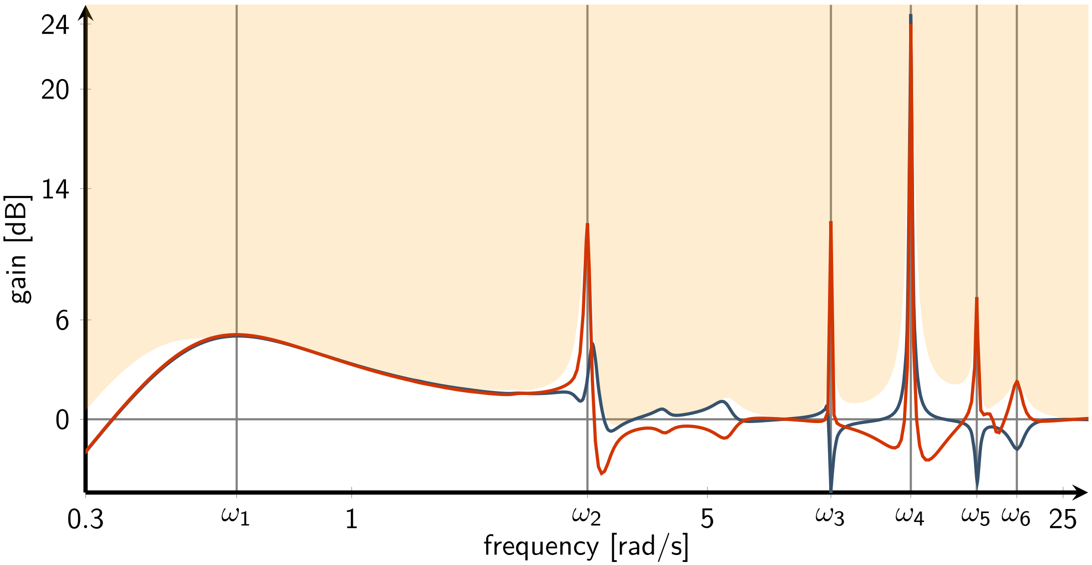
Multi-frequency worst-case



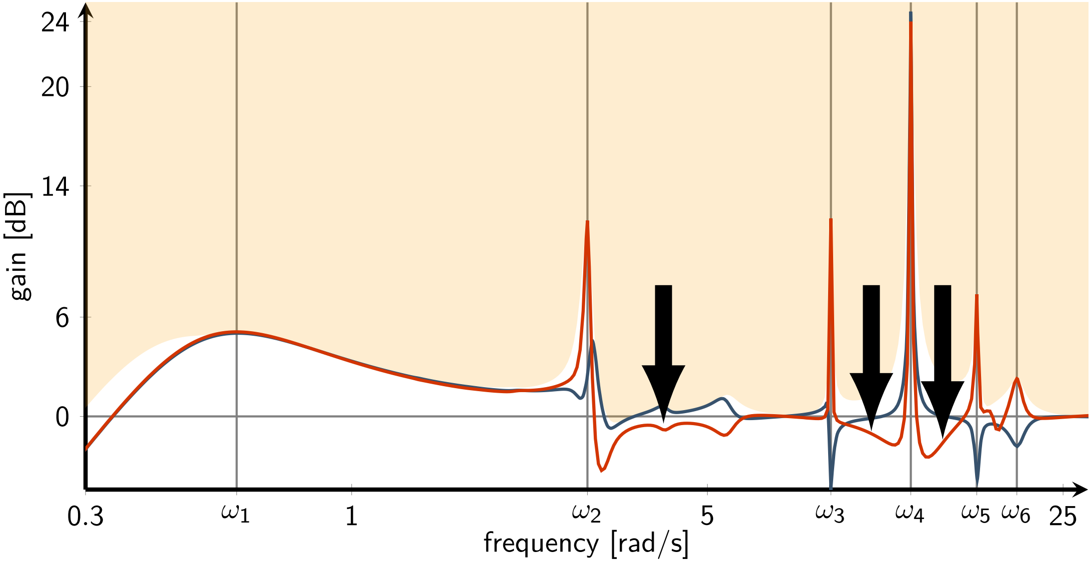
Multi-frequency worst-case



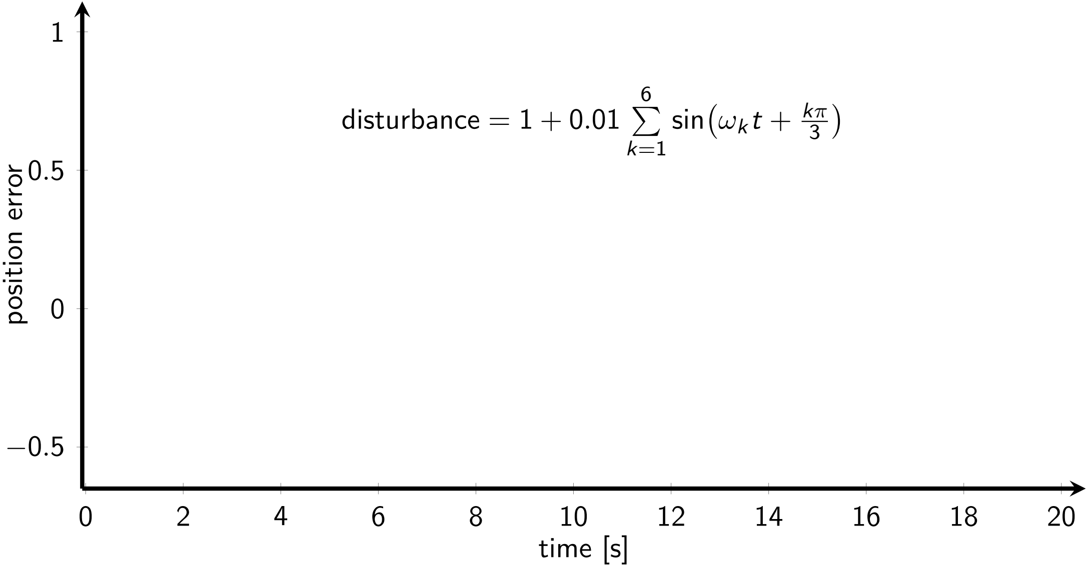
Multi-frequency worst-case



Multi-frequency worst-case

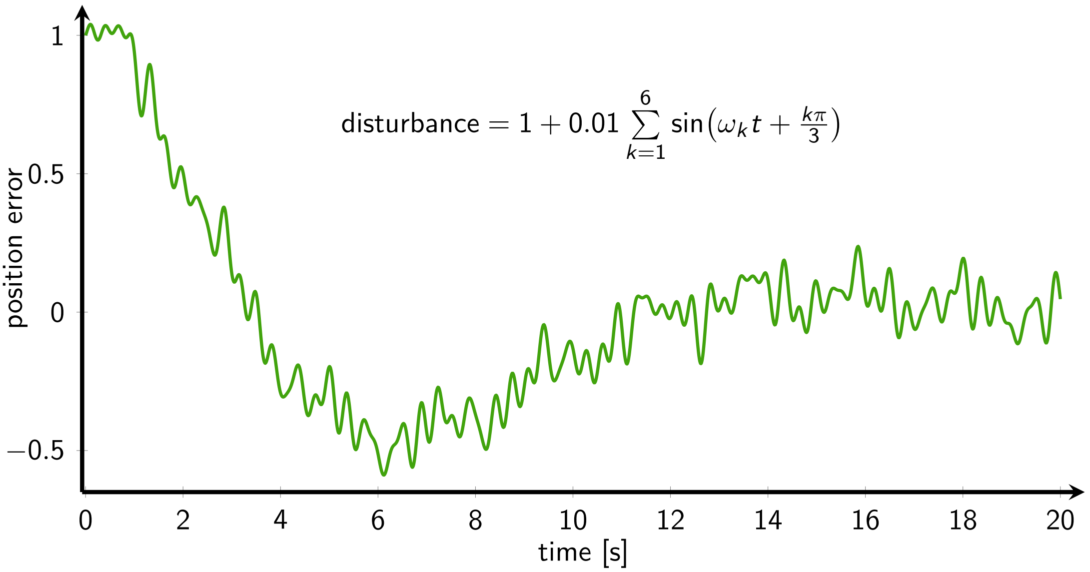


Multi-frequency worst-case

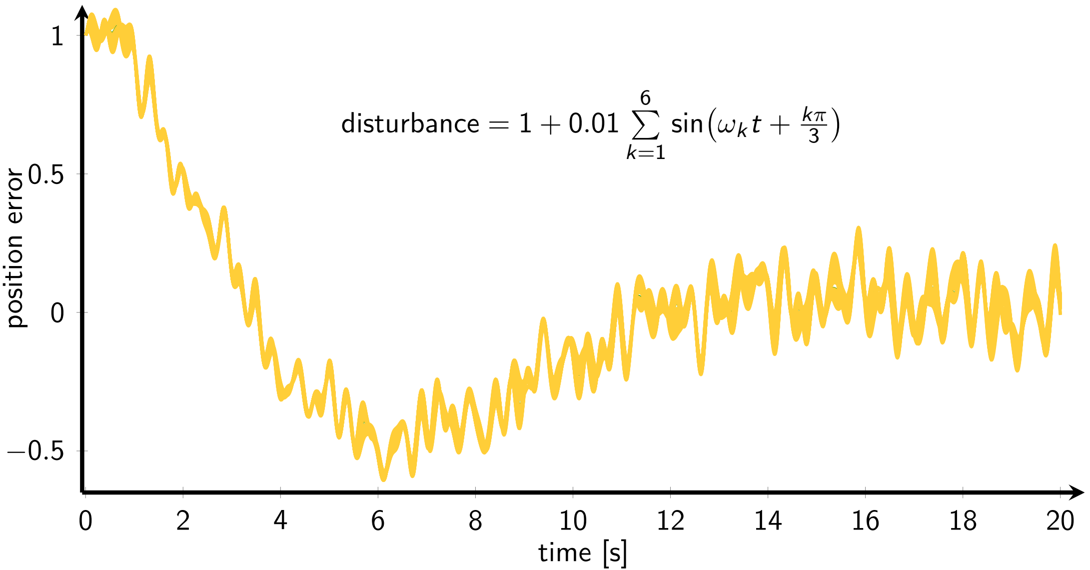


$$\text{disturbance} = 1 + 0.01 \sum_{k=1}^{6} \sin\left(\omega_k t + \frac{k\pi}{3}\right)$$

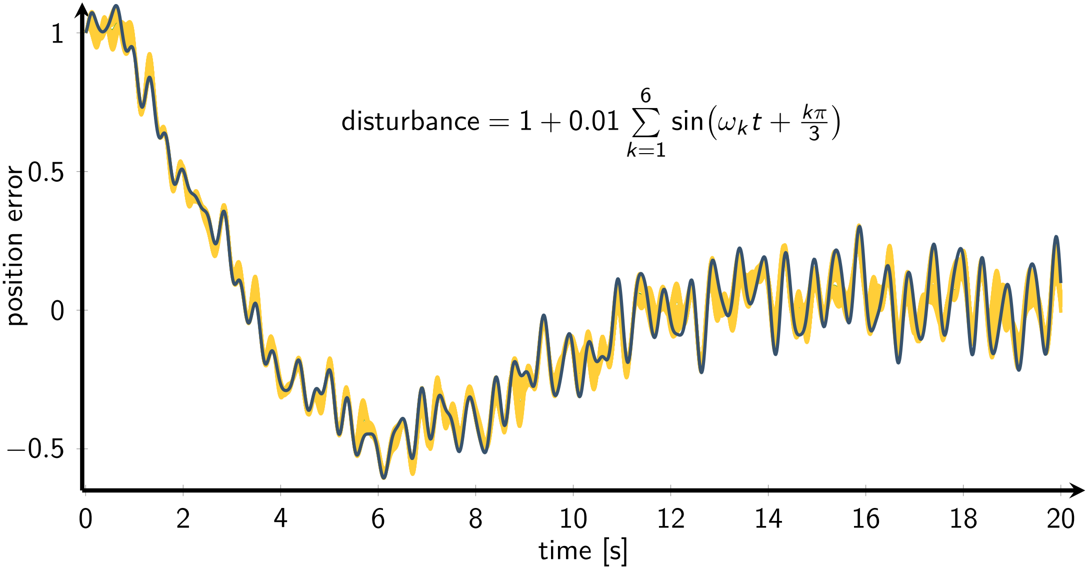
Multi-frequency worst-case



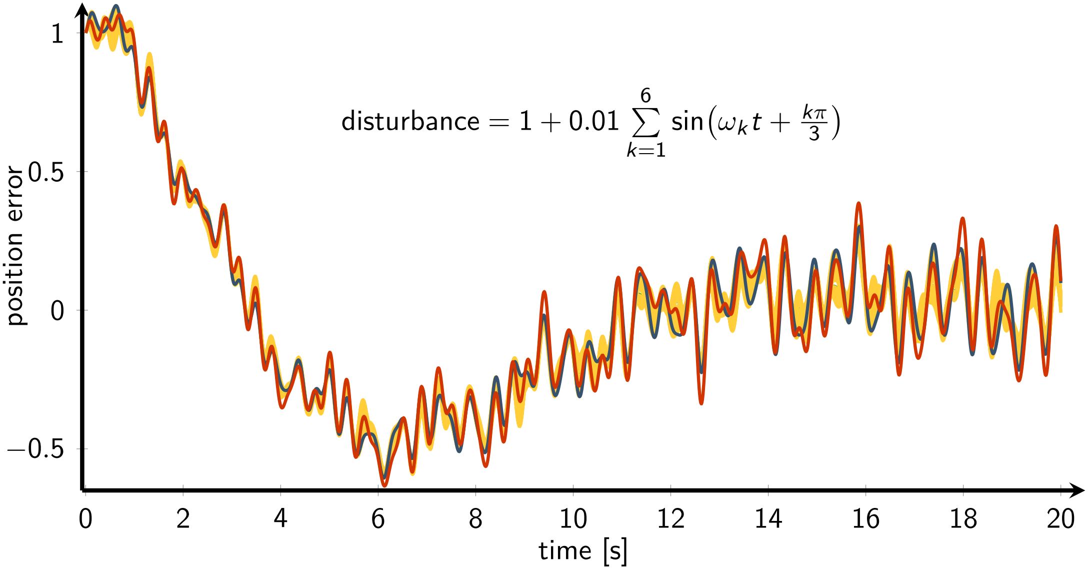
Multi-frequency worst-case



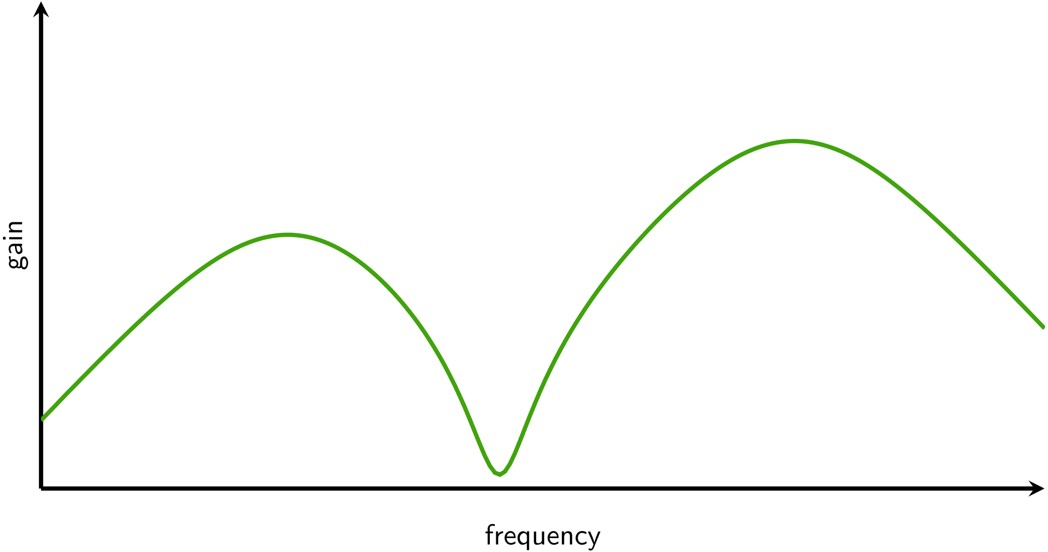
Multi-frequency worst-case



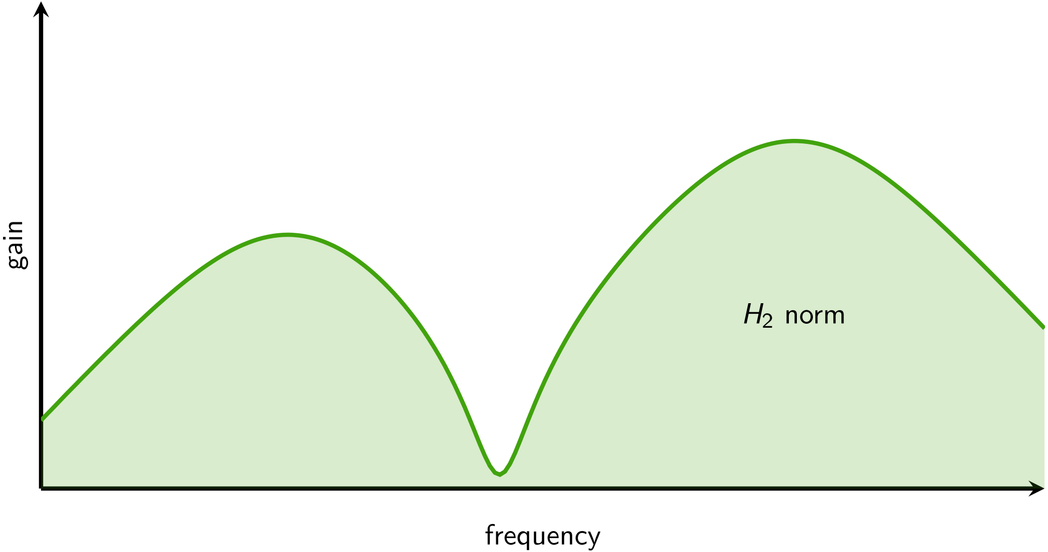
Multi-frequency worst-case



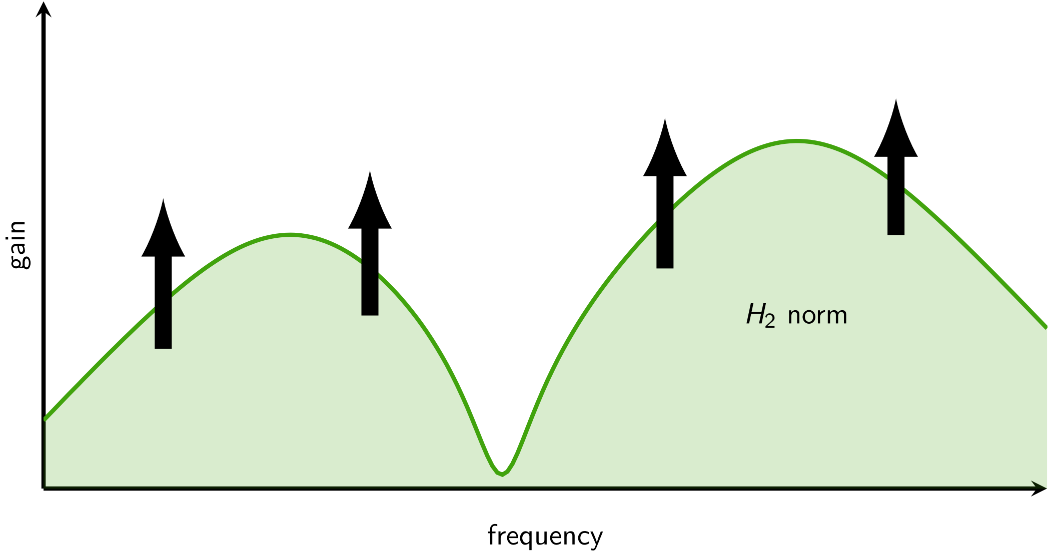
Worst-case H_2 norm



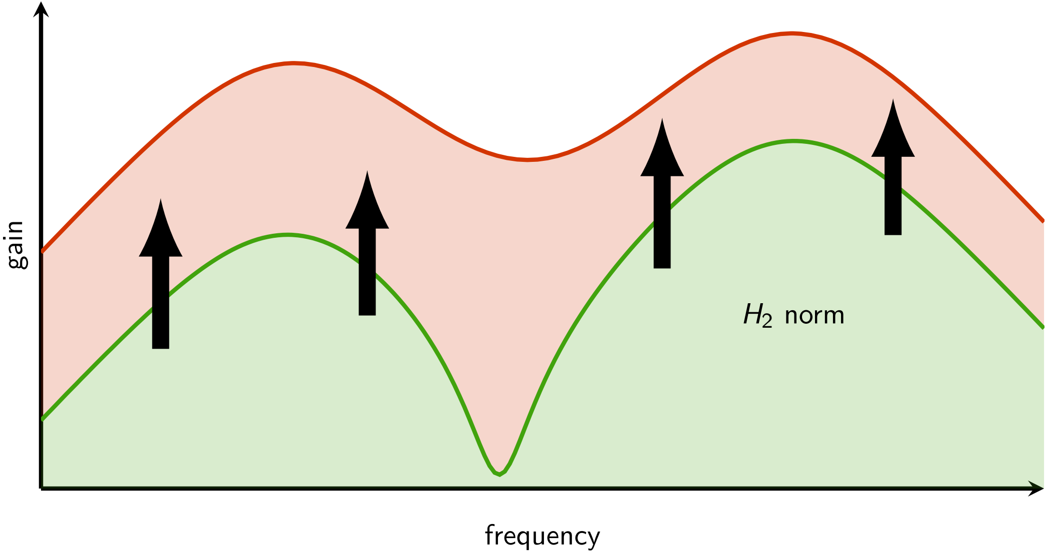
Worst-case H_2 norm



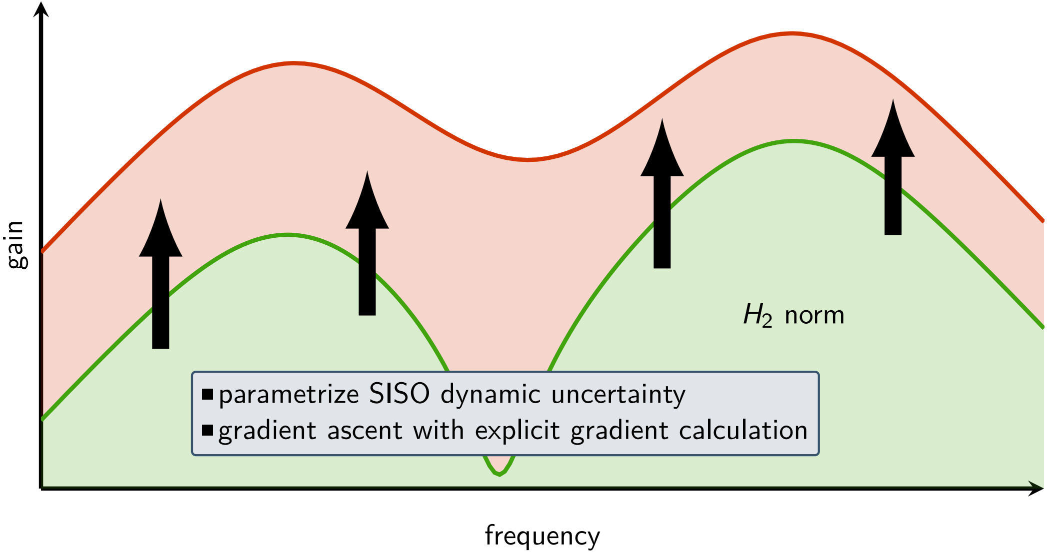
Worst-case H_2 norm



Worst-case H_2 norm

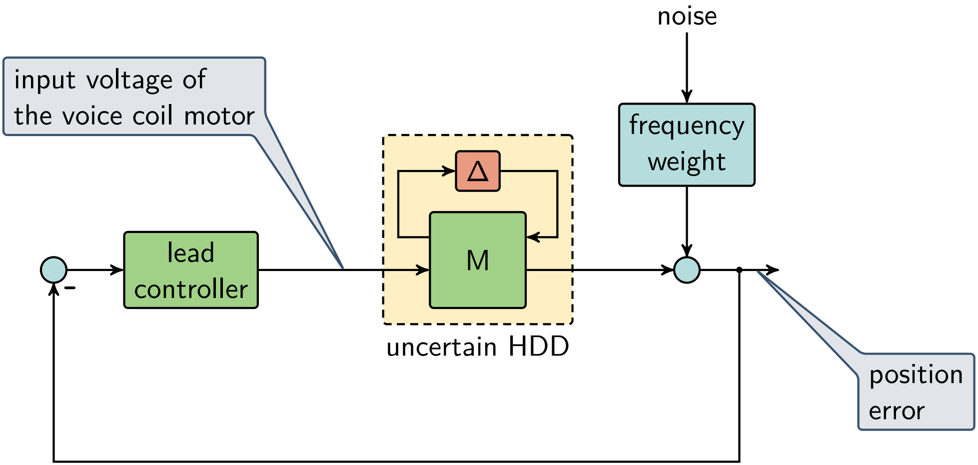


Worst-case H_2 norm

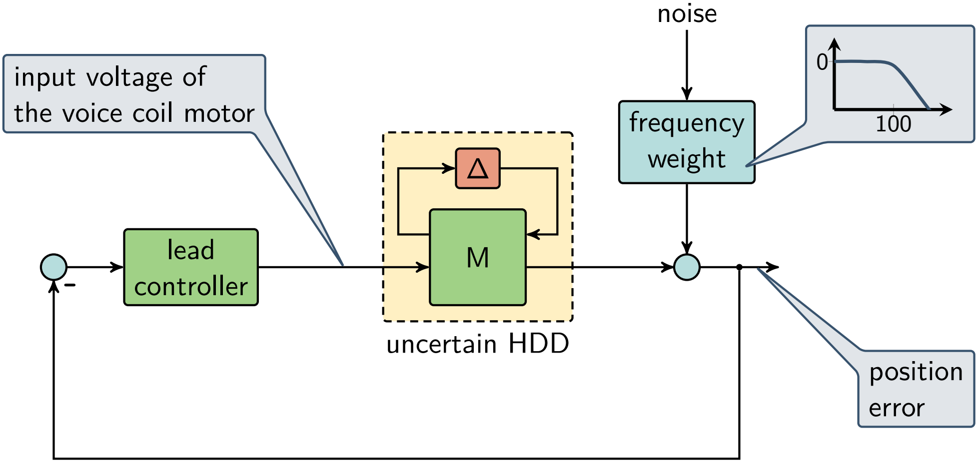


- parametrize SISO dynamic uncertainty
- gradient ascent with explicit gradient calculation

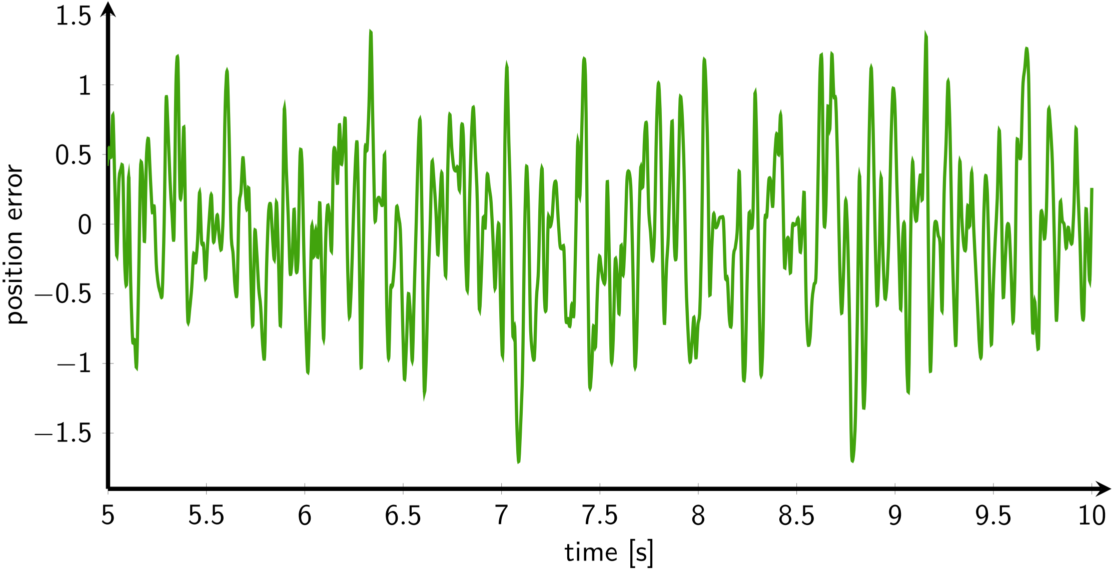
Worst-case H_2 performance



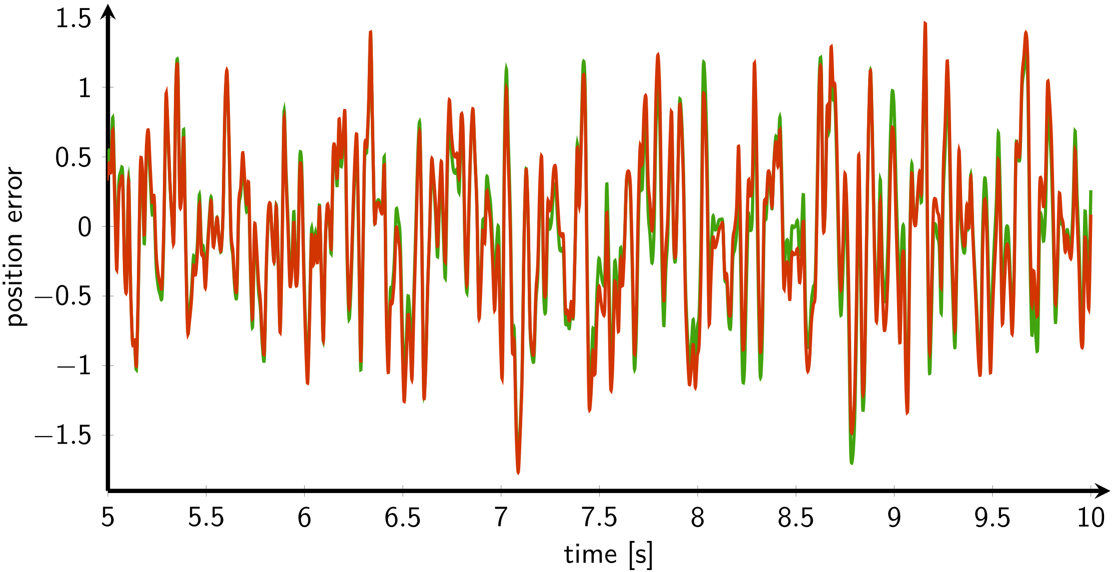
Worst-case H_2 performance



Worst-case H_2 performance



Worst-case H_2 performance



Worst-case H_2 performance

	H_2 norm
nominal	
1000 samples avg.	
1000 samples max.	
worst-case	

Worst-case H_2 performance

	H_2 norm
nominal	**7.11**
1000 samples avg.	
1000 samples max.	
worst-case	

Worst-case H_2 performance

	H_2 norm
nominal	**7.11**
1000 samples avg.	**7.12**
1000 samples max.	
worst-case	

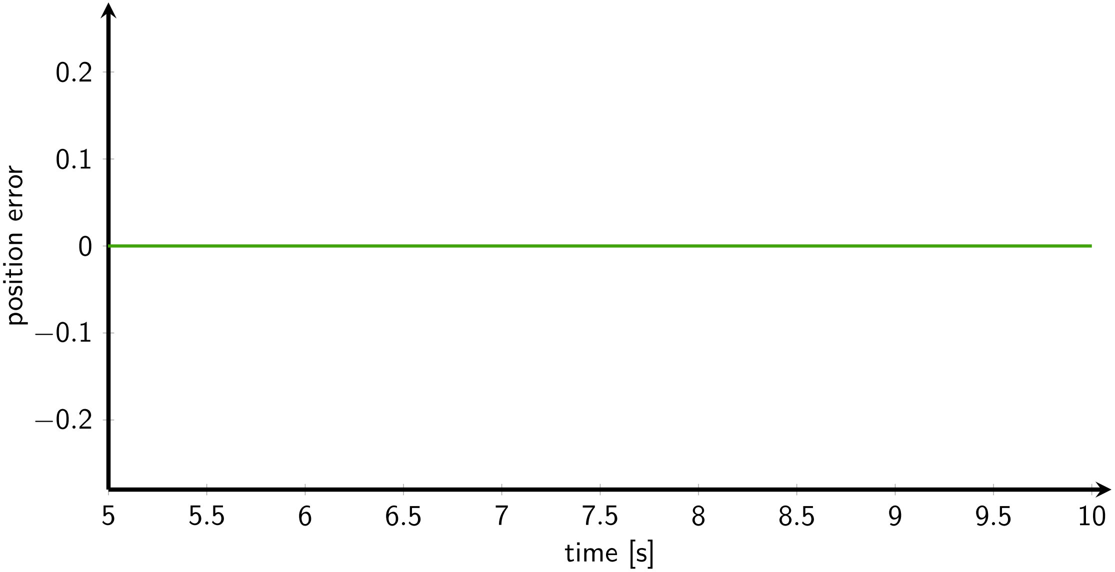
Worst-case H_2 performance

	H_2 norm
nominal	**7.11**
1000 samples avg.	**7.12**
1000 samples max.	**7.32**
worst-case	

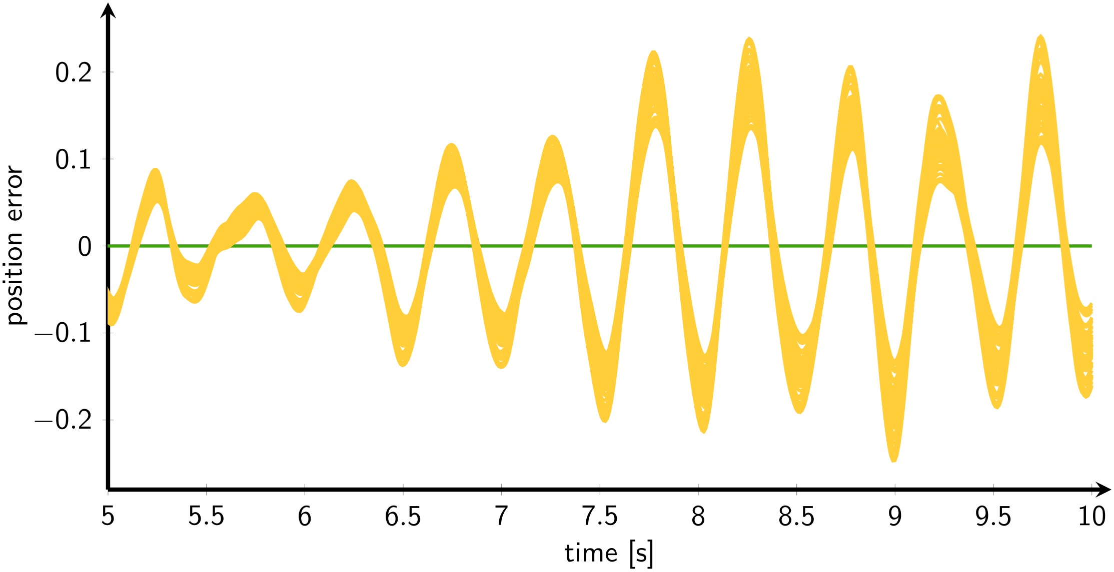
Worst-case H_2 performance

	H_2 norm
nominal	**7.11**
1000 samples avg.	**7.12**
1000 samples max.	**7.32**
worst-case	**7.42**

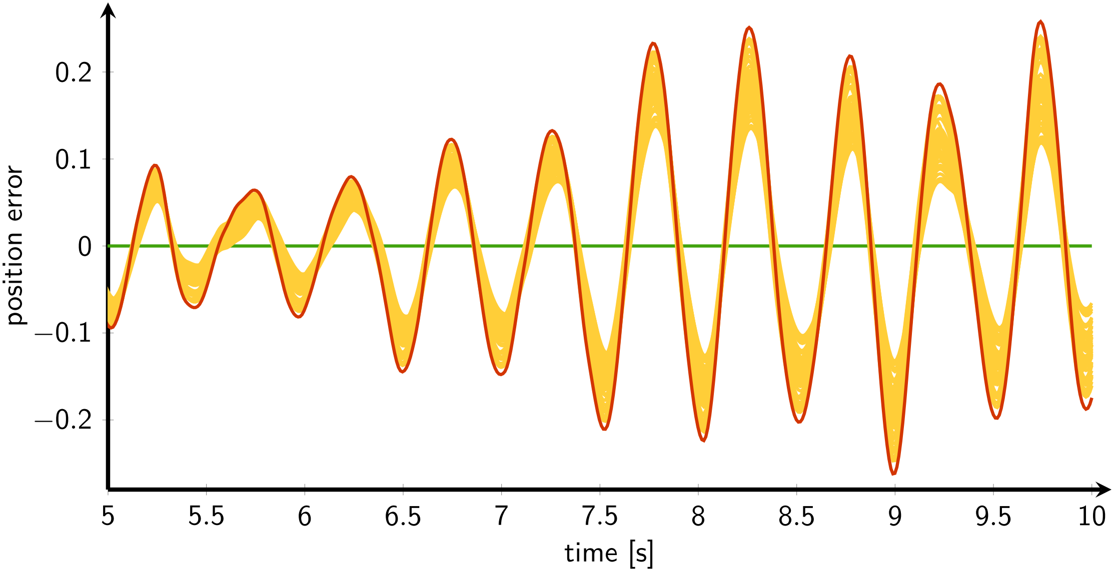
Worst-case H_2 performance



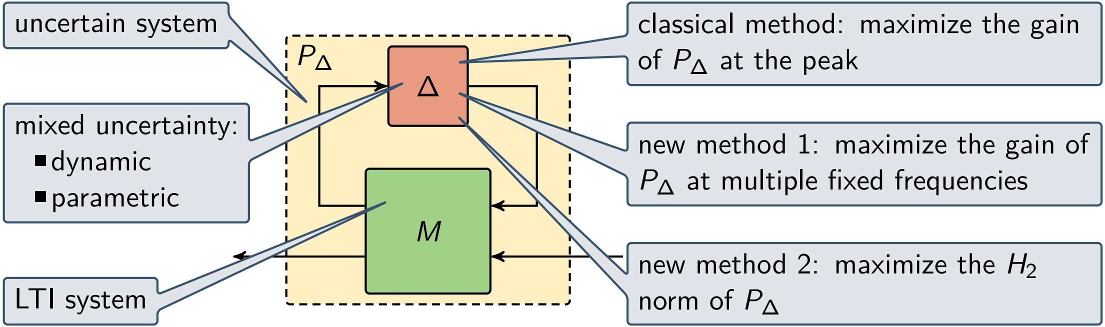
Worst-case H_2 performance



Worst-case H_2 performance



Worst-case uncertainty construction



- Patartics, Seiler, Vanek. *Construction of an uncertainty to maximize the gain at multiple frequencies.* ACC 2020.
- Patartics, Seiler, Takarics, Vanek. *Worst-case uncertainty construction via multifrequency gain maximization.* Submitted to the TCST, 2020

Practical considerations for industrial use

Given:

1. High fidelity, nonlinear simulation
   a) Environmental/sensor noise
   b) Uncertain parameters
2. Certification requirements

Analysis workflow:

1. Linearization including model uncertainties
   a) Construction of this model from nonlinear simulation is not completely automated and often requires domain expertise.
2. Linear analyses
   a) Further development of the algorithm for numerical reliability
   b) Extension of the algorithm to more general uncertainties, e.g. delays, sector-bounded nonlinearities.
   c) Compare different metrics (H_∞, multi-frequency, H_2) for their engineering relevance.
3. Monte Carlo simulations on high fidelity model
   a) Study "bad" uncertainties from linear analysis to complement random samples. Generate several worst-cases based on different assumptions, e.g. frequency range of disturbances.